## SUBDIVISION SCHEMES FOR THE PARAMETERIZATION OF CONTACT SURFACES WITH ARBITRARY MESH TOPOLOGY

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By the use of a  $C^0$ -continuous surface, which occur when the interacting bodies are discretized by facet elements, the enforcement of contact constraints may lead to a number of well known problems such as non-physical oscillations of contact forces, contact cycling and loss of quadratic convergence. In the recent paper [3] we showed that higher levels of continuity of the surface parameterization can avoid these problems. However, previously developed parameterization techniques can only deal with regular mesh topologies (four quadrilaterals or six triangles that meet in one node, see [1] among others), or are unable to accomplish  $C^1$ -continuity everywhere. For our approach we use subdivision schemes for the surface representation, which attain  $C^2$ -continuity everywhere except at *irregular nodes* (i.e. more or less than four quadrilaterals or six triangles meet in one node), where still  $C^1$ -continuity is preserved [3].

We discuss the features of four subdivision schemes when used for the parameterization of contact surfaces and show how they can be evaluated analytically, which is a necessary requirement for rapid evaluation of the element arrays associated with the finite-element discretization. In particular, we develop a frictional contact element for adaptively refined meshes (with or without hanging nodes). It avoids the need for an interpolation because it directly uses the nodes of the subdivision scheme to represent its degrees of freedom. We call it a *composite* (as opposed to *overlay*) *contact element;* since this formulation enables that the geometrical model and the finite-element analysis can be based on an identical representational paradigm (IRP). This means that both the deformed and the undeformed geometries are supported by the subdivision scheme, which is desirable for the link between the design and the analysis of the interacting bodies [2]. Hence, exactly the same surfaces that are designed will be used for the finite-element analysis, which is crucial for the contact interaction. Therefore, the quality of the surface representation is independent from the mesh density.

These properties are favorable so that this approach turns out to be attractive for the contact simulation of bodies with complex surface topology, as frequently encountered for biomechanical problems. The performance of the proposed algorithm and the accuracy are demonstrated by means of a representative example for which analytical solutions are available. The advantages of IRP are demonstrated using biomechanical applications.

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## References

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